Indian Statistical Institute
Back Paper
Topology IV - MMath II

Max Marks: 100 Time: 180 minutes.

Throughout, U, $V \ldots$ will denote open subsets in some Euclidean space.

(1) Show that there exist isomorphisms
$$\mathbb{R}^3 \xrightarrow{i} \mathrm{Alt}^1(\mathbb{R}^3), \mathbb{R}^3 \xrightarrow{j} \mathrm{Alt}^2(\mathbb{R}^3)$$
given by $i(v)(w) =< v, w >$ and $j(v)(w_1, w_2) = \det(v, w_1, w_2)$. Here $< -, - >$ denotes the usual inner product on $\mathbb{R}^3$. [10]

(2) Construct the de Rham complex $\Omega^*(U)$ of U. Verify that this is a cochain complex. [20]

(3) Compute $H^0(U)$. [10]

(4) Let $A \subseteq \mathbb{R}^n$ be a closed subset. Let $R : \mathbb{R}^{n+1} - A \longrightarrow \mathbb{R}^{n+1} - A$ denote the reflection $(x_1, \ldots, x_n) \mapsto (x_1, \ldots, x_n, -x_{n+1})$. Show that the induced homomorphism
$$R^* : H^{p+1}(R^{n+1} - A) \longrightarrow H^{p+1}(R^{n+1} - A)$$
is multiplication by -1 for all $p \geq 0$. [15]

(5) Given a closed subset $A \subseteq \mathbb{R}^n$ and distinct points $p, q \in \mathbb{R}^n$ we say that A separates p from q if p and q belong to different components of $\mathbb{R}^n - A$. Let A and B be two disjoint closed subsets of $\mathbb{R}^n$. Given two distinct points p, q in $\mathbb{R}^n - (A \cup B)$ show that if neither A nor B separates p from q, then $A \cup B$ does not separate p from q. [15]

(6) What is an orientation form on a manifold? If ω_i, $i = 1, 2$ is an orientation form on the manifold M_i, $i = 1, 2$ and $\pi_i : M_1 \times M_2 \longrightarrow M_i$, $i = 1, 2$ is the projection, then show that $\pi_1^*\omega_1 \wedge \pi_2^*\omega_2$ is an orientation form on $M_1 \times M_2$. [10]

(7) Let $n \geq 2$. Compute $H^p(\mathbb{R}P^{n-1})$, $p \geq 0$. [20]